TM5 PR 2.8

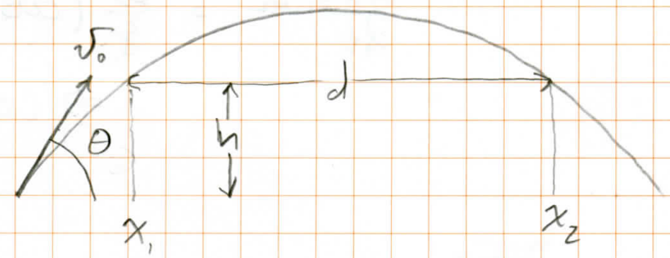
TM 5 2-8 SHOW THAT THE DISTANCE BETWEEN TWO POINTS ON A TRAJECTORY AT THE SAME HEIGHT IS (θ FOR MAX RANGE)

$$d = \frac{v_0}{g}\sqrt{v_0^2 - 4gh}$$

USE KINEMATICS

$$x = (v_0 \cos\theta)t$$

$$y = (v_0 \sin\theta)t - \tfrac{1}{2}gt^2$$



FIND THE DISTANCE BETWEEN x_1 AND x_2 AT $y = h$

$$h = (v_0 \sin\theta)t - \tfrac{1}{2}gt^2 \Rightarrow \tfrac{1}{2}gt^2 - (v_0 \sin\theta)t + h = 0$$

SOLVE FOR t

$$t = \frac{v_0 \sin\theta \pm \sqrt{v_0^2 \sin^2\theta - 4(\frac{1}{2}g)h}}{g}$$

THE TWO DISTANCES ARE THEN

$$x_1 = \frac{v_0 \cos\theta}{g}\left(v_0 \sin\theta - \sqrt{v_0^2 \sin^2\theta - 2gh}\right)$$

$$x_2 = \frac{v_0 \cos\theta}{g}\left(v_0 \sin\theta + \sqrt{v_0^2 \sin^2\theta - 2gh}\right)$$

FIND THE DIFFERENCE

$$d = x_2 - x_1 = \frac{v_0 \cos\theta}{g}\left(2\sqrt{v_0^2 \sin^2\theta - 2gh}\right)$$

FIND θ FOR THE MAXIMUM RANGE

$$\text{RANGE} = x_{FINAL} = (v_0 \cos\theta)\, t_{FLIGHT}$$

$$\cancelto{0}{y_{FINAL}} = \cancelto{0}{y_0} + (v_0 \sin\theta)\, t_{FLIGHT} - \tfrac{1}{2}g\, t^2_{FLIGHT}$$

$$t_{FLIGHT} = \frac{2v_0 \sin\theta}{g}$$

NOTE BOOK

Miquelrius

$$x_{FINAL} = \frac{2v_0^2 \sin\theta \cos\theta}{g} = \frac{v_0^2 \sin 2\theta}{g}$$

FIND THE MAXIMUM

$$\frac{dx_{FINAL}}{dt} = \frac{v_0^2}{g}(\cos 2\theta)(2) = 0$$

$$\Rightarrow \cos 2\theta = 0$$

$$\Rightarrow \theta = \frac{\pi}{4} = 45^0$$

THUS

$$d = \frac{2v_0 \cos\theta}{g}\sqrt{v_0^2 \sin^2\theta - 2gh}$$

SET $\cos\theta = \sin\theta = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$$d = \frac{v_0}{g}\left(\frac{\cancel{2}\sqrt{2}}{\cancel{2}}\right)\sqrt{\frac{v_0^2}{2} - 2gh} = \frac{v_0}{g}\sqrt{\frac{\cancel{2}v_0^2}{\cancel{2}} - 4gh}$$

$$\boxed{d = \frac{v_0}{g}\sqrt{v_0^2 - 4gh}}$$